FTD-TT- 62-1623

TRANSLATION

STRESSED STATE OF CYLINDRICAL SHELLS, REINFORCED WITH RIBS

By

D. V. Vaynberg, V. O. Zaruts'kiy and B. Z. Itenberg

FOREIGN TECHNOLOGY DIVISION

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FTD-TT- 62-1623/1+2

UNEDITED ROUGH DRAFT TRANSLATION

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English Pages: 12

SOURCE: Ukrainian Periodical, Prikladnaya Mekhanika, Vol. 6, Nr. 4, 1960, pp 375-384

S/198-60-6-4

PREPARED BY:

TRANSLATION DIVISION
FOREIGN TECHNOLOGY DIVISION
WP-AFB, OHIO.

FTD-TT- 62-1623/1+2

Date 30 April 19 63

Stressed State of Cylindrical Shells, Reinforced with Ribs

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Arrangement of the problem. In the given report are described the methods of investigating the stressed state of a circular cylindrical shell, reinforced with a regular screen of annular and rectilinear ribs. The effect of constant intensity pressure and longitudinal meridional forces on the sheel has been investigated.

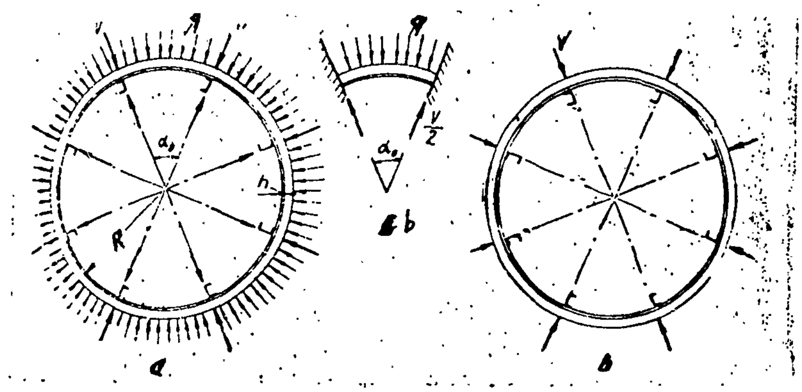

Fig.1.

Selection of the calculation method depends upon the correlation of basic parameters of the system: closeness of placing ribs and relative rigidity of shell walls and ribs.

A sufficiently large number of ribs allows to average the elastic qualities of the system and consider same from the position of theory of thin structurally-orthotropic shells of revolution. Concentration of forces in the zone of rib arrangement, in analogy with the problem of ribbed plate [1, 2] can be determined in approximation on the basis of perturbation theory.

At an average number of ribs is recommended a different approach to solving the problem, based on the utilization of deformation characteristics of symmetrical systems. The stressed state of a ribbed regular shell (fig.1,a) can be obtained by applying

two caluulation systems.

In the first one will be discussed a rectangular panel of the shell with rigidly fastened annular and rectilinear edges under the effect of a given load (fig1,b).

In the other one is discussed a given closed shell under the effect of radial forces, applied to the ribs; these forces in magnitude equal to reactions V, found in first calculation, but have an opposite sign in comparison with same (fig.1,c). Calculation of shell in the other case can be realized by approximation methods. Since the rigidity of the ribs is high, a comparison was made with the rigid walls of the shell, it is advisable to change over to the calculated model in form of a spatial cyclic symmetrical frame, formed by rings and meridional ribs. Here can be applied the theory of cyclically symmetrical frames [3]. In the presence of low rigidity ribs calculation of the system in second case can be made with the aid of equations of elastic deformation of structurally-orthotropic shells.

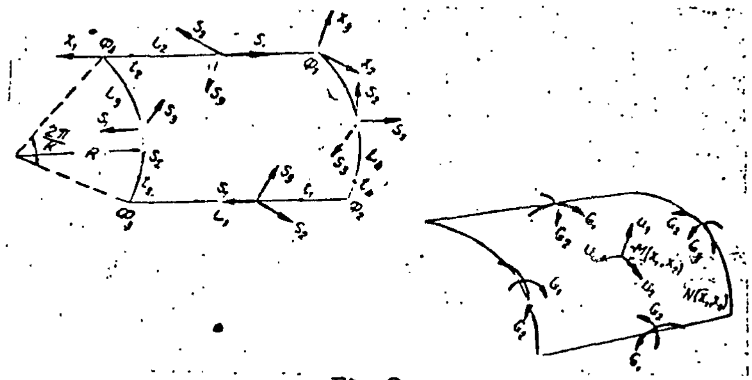

Fig.2

Special consideration should be given to the case of a low number of relatively rigid ribs, when it has been decided to search for an accurate solution of the problem. The problem here is reduced to studying the stressed state of a rectangular panel of a cylindrical shell. The other method is based on the utilization of integro-differential equations, which emanate from the theorem of report mutuality [4,5] in the report by M.O.Kil'chevskiy [6-8].

Par.2. Differential equations of the problem and their solutions. We will write a system of equations of technical theory of shells [9]

$$\begin{aligned}
&\frac{\partial^2 u_1}{\partial \alpha^2} + \frac{1-\nu}{2}\cdot\frac{\partial^2 u_1}{\partial \beta^2} + \frac{1+\nu}{2}\cdot\frac{\partial^2 u_2}{\partial \alpha \partial \beta} + \nu\frac{\partial u_3}{\partial \alpha} = -\frac{R^2}{B}X_1,\\
&\frac{1+\nu}{2}\cdot\frac{\partial^2 u_1}{\partial \alpha \partial \beta} + \frac{\partial^2 u_2}{\partial \beta^2} + \frac{1-\nu}{2}\cdot\frac{\partial^2 u_2}{\partial \alpha^2} + \frac{\partial u_3}{\partial \beta} = -\frac{R^2}{B}X_2,\\
&\nu\frac{\partial u_1}{\partial \alpha} + \frac{\partial u_2}{\partial \beta} + a^2\Delta\Delta u_3 + u_3 = \frac{R^2}{B}X_3,
\end{aligned} \tag{2.1}$$

where u_1, u_2, u_3 - components of displacement vector of the center surface of the shell (fig.2); $\alpha = \frac{x_1}{R}$, $\beta = \frac{x_2}{R}$ - dimensionless coordinates, R - radius of center surface of the shell; h-thickness of shell; $a^2 = \frac{h^2}{12R^2}$; B, ν - tensile strength of shell and Poison coefficient; X_1, X_2, X_3 - components of surface load (accepted as $X_2 = 0$)

We shall examine a cylindrical panel, the rectilinear edge of which (in conformity with the cyclic symmetry of the shell) are connected with the elastically settled ribs, which do not rotate, and curvilinear - hinged. The load consists of normal pressure p and axial meridional forces N_1 appalied to the faces.

In this case

$$\begin{aligned}
X_1 &= -N_1[\delta(\alpha) - \delta(\alpha-\alpha_1)] - T_1(\alpha,0)[\delta(\beta) + \delta(\beta-\beta_1)],\\
X_3 &= p - T_3(\alpha,0)[\delta(\beta) + \delta(\beta-\beta_1)],
\end{aligned} \tag{2.2}$$

where $\beta_1 = \frac{2\pi}{k}$, $\alpha_1 = \frac{l}{R}$; l-length of shell; k-number of ribs. $T_3(\alpha,0) = -T_3(\alpha,\beta_1)$ and $T_1(\alpha,0) = -T_1(\alpha,\beta_1)$ - general (in Kirchhoff understanding) cross sectional and tangent forces.

The law of signs for forces and moments is shown in fig.2. The magnitudes $\delta(\alpha)$; $\delta(\alpha-\alpha_1)$; $\delta(\beta)$; $\delta(\beta-\beta_1)$ are Dirac's delta functions which can also be presented in form of incongruous trigonometric series, e.g.:

$$\delta(\alpha-\alpha_1) = \frac{2}{\alpha_1 R}\left[\frac{1}{2} + \sum_{m=1}^{\infty}\cos\frac{\pi m}{\alpha_1}(\alpha-\alpha_1)\right]. \tag{2.3}$$

On the curvilinear edges of the panel take place boundary conditions

$$\begin{aligned}
&u_2(0,\beta) = u_2(\alpha_1,\beta) = 0, \quad u_3(0,\beta) = u_3(\alpha_1,\beta) = 0,\\
&G_1(0,\beta) = G_1(\alpha_1,\beta) = 0, \quad S_1(0,\beta) = S_1(\alpha_1,\beta) = -N_1.
\end{aligned} \tag{2.4}$$

on rectilinear contours - conditions of cyclic symmetry for displacements of angles of rotation of the shell

$$u_2(\alpha, 0) = u_2(\alpha, \beta_1) = 0, \qquad \left.\frac{\partial u_3(\alpha, \beta)}{\partial \beta}\right|_{\beta=0} = \left.\frac{\partial u_3(\alpha, \beta)}{\partial \beta}\right|_{\beta=\beta_1} = 0 \tag{2.5}$$

and conditions of coupling with elastic ribs.

$$2T_1(\alpha, 0) = -2T_1(\alpha, \beta_1) = -p_1(\alpha), \quad 2T_3(\alpha, 0) = -2T_3(\alpha, \beta_1) = -p_2(\alpha). \tag{2.6}$$

The magnitudes p_1 (α) and p_2 (α) are designated by rib displacements, which are equal to the displacements of corresponding contour points of the shell

$$p_1(\alpha) = \frac{E_1 F_1}{R^2} \cdot \left.\frac{\partial^2 u_1(\alpha, \beta)}{\partial \alpha^2}\right|_{\beta=0}, \quad p_2(\alpha) = -\frac{E_1 I_1}{R^4} \cdot \left.\frac{\partial^4 u_3(\alpha, \beta)}{\partial \alpha^4}\right|_{\beta=0}, \tag{2.7}$$

where F_1, I_1 - area and moment of inertia o transverse cross section of rib; E_1 - Young modulus of the rib material. The values $T_1(\alpha, 0)$ and T_3 $(\alpha, 0)$ are represented by trigonometric series with unknown coefficients a_m and b_m

$$T_1(\alpha, 0) = \sum_{m=1}^{\infty} b_m \cos \frac{\pi m \alpha}{\alpha_1}, \qquad T_3(\alpha, 0) = \sum_{m=1}^{\infty} a_m \sin \frac{\pi m \alpha}{\alpha_1}. \tag{2.8}$$

Solution of the system of equations (2.1) for load (2.2) in conformity with boundary conditions (2.4) and (2.5) are sought in the formula

$$\begin{aligned} u_1 &= \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} A_{mn} \cos \frac{\pi m \alpha}{\alpha_1} \cos \frac{\pi n \beta}{\beta_1}, \\ u_2 &= \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} B_{mn} \sin \frac{\pi m \alpha}{\alpha_1} \sin \frac{\pi n \beta}{\beta_1}, \\ u_3 &= \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} C_{mn} \sin \frac{\pi m \alpha}{\alpha_1} \cos \frac{\pi n \beta}{\beta_1}. \end{aligned} \tag{2.9}$$

Utilizing (2,3), (2,6), (2,7), (2,8) we find

$$\begin{aligned} A_{mn} &= A_{m0}\delta_{0n} + a_m A'_{mn} + b_m A''_{mn}, \\ B_{mn} &= a_m B'_{mn} + b_m B''_{mn}, \\ C_{mn} &= C_{m0}\delta_{0n} + a_m C'_{mn} + b_m C''_{mn}, \end{aligned} \tag{2.10}$$

where

For equation 2,11 see page 4a

$$
A_{m0} = \frac{2R^4}{D} \cdot \frac{1 - (-1)^m}{m^2 (m^4 + t)} \cdot \frac{\alpha_1^6}{\pi^6} \left[\nu p + \frac{N_1}{R} \left(1 + \frac{1 - \nu^2}{t} m^4 \right) \right],
$$

$$
A'_{mn} = C'_{mn} = \frac{R^3}{D} \cdot \frac{2\alpha_1^5}{\pi^5 \beta_1} \cdot \frac{1 + (-1)^n}{1 + \delta_{0n}} \cdot \frac{m (\gamma^2 n^2 - m^2 \nu)}{(m^2 + \gamma^2 n^2)^4 + t m^4},
$$

$$
A''_{mn} = -\frac{R^3}{D} \cdot \frac{2\alpha_1^6}{\pi^6 \beta_1} \cdot \frac{1 + (-1)^n}{1 + \delta_{0n}} \cdot \frac{m^2 + \frac{1 - \nu^2}{t} (m^2 + \gamma^2 n^2)^2 \left(m^2 + \frac{2\gamma^2}{1 - \nu} n^2 \right)}{(m^2 + \gamma^2 n^2)^4 + m^4 t},
$$

$$
B'_{mn} = -\frac{R^3}{D} \frac{2\alpha_1^6}{\pi^5 \beta_1^2} \cdot \frac{1 + (-1)^n}{1 + \delta_{0n}} \cdot \frac{n [n^2 \gamma^2 + (2 + \nu) m^2]}{(m^2 + \gamma^2 n^2)^4 + m^4 t}, \qquad (2.11)
$$

$$
B''_{mn} = -\frac{R^3}{D} \cdot \frac{2\alpha_1^7}{\pi^6 \beta_1^2} \cdot \frac{1 + (-1)^n}{1 + \delta_{0n}} \cdot \frac{mn \left[1 + \frac{(1 + \nu)^2}{t} (m^2 + \gamma^2 n^2)^2 \right]}{(m^2 + \gamma^2 n^2)^4 + m^4 t},
$$

$$
C_{m0} = \frac{R^4}{D} \cdot \frac{2\alpha_1^4}{\pi^5} \cdot \frac{1 - (-1)^m}{m (m^4 + t)} \left[p + \nu \frac{N_1}{R} \right],
$$

$$
C''_{mn} = -\frac{R^3}{D} \cdot \frac{2\alpha_1^4}{\pi^4 \beta_1} \cdot \frac{1 + (-1)^n}{1 + \delta_{0n}} \cdot \frac{(m^2 + \gamma^2 n^2)^2}{(m^2 + \gamma^2 n^2)^4 + m^4 t}.
$$

Here gamma = $\frac{\alpha_1}{\beta_1}$, D = bending rigidity of shell;

$$t = \frac{1-\nu^2}{a^2} \cdot \frac{\alpha_1^4}{\pi^4}, \qquad \delta_{0n} = \begin{cases} 1, & \text{якщо } n = 0, \\ 0, & \text{якщо } n \neq 0. \end{cases}$$ 2.11a

In accordance with the boundary conditions (2.6) and (2.7) we obtain a system of equations to designate a_m and b_m

$$b_m = \frac{\pi^2 m^2}{\alpha_1^2}(A_{m0} + a_m A'_m + b_m A^*_m)\frac{E_1 F_1}{2R^2},$$

$$a_m = \frac{\pi^4 m^4}{\alpha_1^4}(C_{m0} + a_m C'_m + b_m C^*_m)\frac{E_1 I_1}{2R^4}. \quad (2.12)$$

Here

$$A'_m = C^*_m = \sum_{n=0}^{\infty} A'_{mn}, \quad A^*_m = \sum_{n=0}^{\infty} A^*_{mn}, \quad C'_m = \sum_{n=0}^{\infty} C'_{mn}. \quad (2.13)$$

in case $N_i = 0$ we assume $b_m = 0$. Then

$$a_m = p\frac{m^3\beta_1}{\pi(m^4+t)} \cdot \frac{1-(-1)^m}{1+\psi m^4 S_m(0)}\psi, \quad (2.14)$$

We will write expressions of shell displacement coefficients

$$u_1(\alpha,\beta) = \Phi_1 \sum_{m=1}^{\infty} \frac{1-(-1)^m}{m^2(m^4+t)}\left[\nu + \frac{\psi m^4 S^*_m(\beta)}{1+\psi m^4 S_m(0)}\right]\cos\frac{\pi m\alpha}{\alpha_1},$$

$$u_2(\alpha,\beta) = -\Phi_1\psi\gamma \sum_{m=1}^{\infty} \frac{1-(-1)^m}{m^4+t} \cdot \frac{m^3 S^{**}_m(\beta)}{1+\psi m^4 S_m(0)}\sin\frac{\pi m\alpha}{\alpha_1}, \quad (2.15)$$

$$u_3(\alpha,\beta) = \Phi_2 \sum_{m=1}^{\infty} \frac{1-(-1)^m}{m(m^4+t)}\left[1 - \frac{\psi m^4 S_m(\beta)}{1+\psi m^4 S_m(0)}\right]\sin\frac{\pi m\alpha}{\alpha_1},$$

$$\Phi_1 = \frac{R^4 p}{D} \cdot \frac{2\alpha_1^5}{\pi^6}, \quad \Phi_2 = \frac{R^4 p}{D} \cdot \frac{2\alpha_1^4}{\pi^5}, \quad \psi = \frac{D_1}{D}, \quad D_1 = \frac{E_1 I_1}{R\beta_1},$$

$$S_m(\beta) = \sum_{n=0}^{\infty} \frac{1+(-1)^n}{1+\delta_{0n}} \cdot \frac{(m^2+\gamma^2 n^2)^2}{(m^2+\gamma^2 n^2)^4 + m^4 t}\cos\frac{\pi n\beta}{\beta_1},$$

$$S^*_m(\beta) = m^2 \sum_{n=0}^{\infty} \frac{1+(-1)^n}{1+\delta_{0n}} \cdot \frac{\gamma^2 n^2 - \nu m^2}{(m^2+\gamma^2 n^2)^4 + m^4 t}\cos\frac{\pi n\beta}{\beta_1}, \quad (2.16)$$

$$S^{**}_m(\beta) = \sum_{n=1}^{\infty} \frac{1+(-1)^n}{1+\delta_{0n}} \cdot \frac{n[\gamma^2 n^2 + (2+\nu)m^2]}{(m^2+\gamma^2 n^2)^4 + m^4 t}\sin\frac{\pi n\beta}{\beta_1}.$$

A numerical calculation of shell was made, which has eight longitudinal ribs (k=8) at such data $\frac{l}{R} = 1, \quad \frac{R}{h} = 250, \quad \nu = \frac{1}{3}, \quad E = E_1.$ 2.16a

Results of calculating girders are given in table for such characteristic values

of relative rigidity: $\psi = 0$ (case of absence of longitudinal ribs); $\psi = 4.8$; $\psi = 10$; $\psi = \infty$ (case of rigid fastening rectilinear edges of panels). To facilitate calculations with the exception of expression (2.16) and we will show in closed form a quite congruent part, which corresponds to deformation of the plate. Then when calculating displacements it is possible to measure by five-six members of series according to n and seven-eight members of series according to m.

Par 3. Integro-differential equations and solution of same. The stressed state of a ribbed cylindrical shell can also be determined as result of solving a system of integro-differential equations.

Table

ψ \ α	$\frac{\alpha_1}{2}$	$\frac{\alpha_1}{4}$	$\frac{\alpha_1}{10}$
	$\beta = \frac{\beta_1}{2}$		
0	$0{,}150 \cdot 10^{-5}\frac{pR^4}{D}$	$0{,}150 \cdot 10^{-5}\frac{pR^4}{D}$	$0{,}150 \cdot 10^{-5}\frac{pR^4}{D}$
4,8	$0{,}155 \cdot 10^{-5}\frac{pR^4}{D}$	$0{,}150 \cdot 10^{-5}\frac{pR^4}{D}$	$0{,}154 \cdot 10^{-5}\frac{pR^4}{D}$
10	$0{,}163 \cdot 10^{-5}\frac{pR^4}{D}$	$0{,}154 \cdot 10^{-5}\frac{pR^4}{D}$	$0{,}155 \cdot 10^{-5}\frac{pR^4}{D}$
∞	$0{,}259 \cdot 10^{-5}\frac{pR^4}{D}$	$0{,}220 \cdot 10^{-5}\frac{pR^4}{D}$	$0{,}183 \cdot 10^{-5}\frac{pR^4}{D}$
	$\beta = 0$		
0	$0{,}150 \cdot 10^{-5}\frac{pR^4}{D}$	$0{,}150 \cdot 10^{-5}\frac{pR^4}{D}$	$0{,}150 \cdot 10^{-5}\frac{pR^4}{D}$
4,8	$0{,}149 \cdot 10^{-5}\frac{pR^4}{D}$	$0{,}139 \cdot 10^{-5}\frac{pR^4}{D}$	$0{,}086 \cdot 10^{-5}\frac{pR^4}{D}$
10	$0{,}146 \cdot 10^{-5}\frac{pR^4}{D}$	$0{,}125 \cdot 10^{-5}\frac{pR^4}{D}$	$0{,}069 \cdot 10^{-5}\frac{pR^4}{D}$
∞	0	0	0

We shall discuss two states of rectangular panel, separated from cylindrical shell.

The basic state appears to be a system which consists of given load $p_i(N,M)$ corresponding to it displacements of center surface of the shell $u(i)_j$ (N,M), forces $S(i)_j$ (N,M) and moments $(G(i)_j$ (N,M) on the limited contours.

Here $N(\bar{\alpha},\bar{\beta})$ - point of applying the load ; $M(\alpha,\beta)$ - point in which displacements are established.

The auxiliary state appears to be a system which consists of concentrated force, having a single projection on the axis of the coordinates and applied to point $M(\alpha,\beta)$ by the center surface of the panel, corresponding displacements v(j)i(M,N), forces N(j)i (M,N) and moments H(j)i (M,N) on the limiting contours.

Having applied the work reciprocity principle to the taken basic and auxiliary states of the shell, we formulate a system of integro-differential equations

$$
\begin{aligned}
u_{(i)j}(N,M) = &\iint\limits_{(S)} p_i(N,Q)\, v_{(j)i}(M,Q)\, dS_Q + \\
&+ \int\limits_{(l)} [\bar{S}_{(i)\delta}(N,t)\, v_{(j)\delta}(M,t) + G_{(i)\gamma}(N,t)\, \varphi_{(j)\gamma}(M,t)]\, dl_t - \\
&- \int\limits_{(l)} [N_{(j)\delta}(M,t)\, u_{(i)\delta}(N,t) + H_{(j)\gamma}(M,t)\, \omega_{(i)\gamma}(N,t)]\, dl_t
\end{aligned}
\qquad (3.1)
$$

Here S-center of shell surface; l-limiting contours; Q-point of the center surface; t-point on contour; $i,j,\delta = 1,2,3$; gamma = 1,2.

The double integral is taken over the entire center surface, single ones - over the entire surface of the panel.

The values $\varphi(j)\gamma$ and omega $(i)\gamma$ - angles of rotation for auxiliary and basic states respectively.

We will introduce into (3.1) general tangents and intersecting forces $T(i)\delta$ (N, t) and $Q(i)\delta$ (M,t) on the contours in basic and auxiliary states respectively we will obtain

For equation 3.2 see page 7a

The values gamma$_1$ = 1 correspond to contours $\alpha = 0$, $\alpha = \alpha_1$, and gamma$_2$ - to contours $\beta = 0$ and $\beta = \beta_1$ (fig.2). Expression A_{ij} $(\varphi_{1}u)$ characterizes the function of normal centered forces in angular points φ_{μ} of the shell on corresponding displacements.

$$
\begin{aligned}
u_{(i)j}(N,M) = \iint_{(S)} p_i(N,Q)\, v_{(j)i}(M,Q)\, dS_Q\, + \\
+ \int_{(l)} [T_{(i)\delta}(N,t)\, v_{(j)\delta}(M,t) + G_{(i)\gamma}(N,t)\, \varphi_{(j)\gamma}(M,t)]\, dl_t\, - \\
- \int [Q_{(j)\delta}(M,t)\, u_{(i)\delta}(N,t) + H_{(j)\gamma}(M,t)\, \omega_{(i)\gamma}(N,t)]\, dl_t + A_{ij}.
\end{aligned}
\tag{3.2}
$$

In case the panel is affected by a normal load of constant intensity equations (3.2) can be written in such form:

$$u_i(M) = p\iint\limits_{(S)} v_{(j)3}(M,Q)\,dS_Q + \int\limits_{(l)} [T_\delta(l)\,v_{(j)\delta}(M,t) + G_\gamma(t)\,\varphi_{(j)\gamma}(M,t)]dl_t - \int\limits_{(l)} [Q_{(j)\delta}(M,t)\,u_\delta(t) + H_{(j)\gamma}(M,t)\,\omega_\gamma(t)]\,dl_t + A_i. \tag{3.3}$$

We shall examine a section of cylindrical ribbed cyclic symmetrical shell. The rectilinear edges of the panel are supported against elastically set ribs, which do not turn; curvilinear edges are hinge fastened.

The boundary conditions in basic state will be on curvilinear edges

$$u_2 = u_3 = G_1 = T_1 = 0, \quad \text{якщо } \alpha = 0 \text{ і } \alpha = \alpha_1, \tag{3.4}$$

on rectilinear edges

$$u_3 = \omega_3 = T_1 = 0, \quad \text{якщо } \beta = 0 \text{ і } \beta = \beta_1, \tag{3.5}$$

$$2T_3 = \pm p_2(\alpha), \quad \text{якщо } \beta = 0 \text{ і } \beta = \beta_1. \tag{3.6}$$

The value $p_2(\alpha)$ is determined from (2.7).

We select an auxiliary state, which satisfies the boundary conditions

$$v_{(j)2} = v_{(j)3} = Q_{(j)1} = H_{(j)1} = 0, \quad \text{якщо } \alpha = 0 \text{ і } \alpha = \alpha_1, \tag{3.7}$$

$$v_{(j)2} = \varphi_{(j)2} = Q_{(j)1} = Q_{(j)3} = 0, \quad \text{якщо } \beta = 0 \text{ і } \beta = \beta_1. \tag{3.8}$$

Utilizing (3.4) - (3.8) we will write (3.3) as follows

For equation 3.9 see page 8a

The difference between the obtained integro-differential equations (3.9) and equations formulated by [8,10,11] in connection with the solution of concrete problems lies in the fact, that for the auxiliart state we have chosen the very same cylindrical panel, but under different fastening conditions, while in the mentioned reports as auxiliary state was selected a plate which is the plan of the shell.

Analyzing term (3.9) we will notice that the problem came to a point of determining T_3 on contours L_1 and L_2.

In conformity with the differential equations (2.1) and boundary conditions (3.7) - (3.8) the displacement components in auxiliary state of the panel will be written

$$u_i(M) = p \iint\limits_{(S)} v_{(j)3}(M, Q)\, dS_Q + \int\limits_{(l_1)} T_3(t_1)\, v_{(j)3}(M, t_1)\, dl_{t_1} + \int\limits_{(l_2)} T_3(t_2)\, v_{(j)3}(M, t_2)\, dl_{t_2}. \qquad (3.9)$$

as follows:

$$v_{(1)3} = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} A_{mn} \sin\frac{\pi m\bar{\alpha}}{\alpha_1}\cos\frac{\pi n\bar{\beta}}{\beta_1}\cos\frac{\pi m\alpha}{\alpha_1}\cos\frac{\pi n\beta}{\beta_1},$$

$$v_{(2)3} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} B_{mn} \sin\frac{\pi m\bar{\alpha}}{\alpha_1}\cos\frac{\pi n\bar{\beta}}{\beta_1}\sin\frac{\pi m\alpha}{\alpha_1}\sin\frac{\pi n\beta}{\beta_1},$$

$$v_{(3)3} = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} C_{mn} \sin\frac{\pi m\bar{\alpha}}{\alpha_1}\cos\frac{\pi n\bar{\beta}}{\beta_1}\sin\frac{\pi m\alpha}{\alpha_1}\cos\frac{\pi n\beta}{\beta_1}, \tag{3.10}$$

where

$$C_{mn} = \frac{\pi^4}{\alpha_1^4}(m^2+\gamma^2n^2)^2 D_{mn},$$

$$B_{mn} = -\frac{\pi^3 n}{\alpha_1^2\beta_1}[(2+\nu)m^2+\gamma^2n^2]D_{mn},$$

$$A_{mn} = -\frac{\pi^3 m}{\alpha_1^3}[\gamma^2n^2-\nu m^2]D_{mn}.$$

$$D_{mn} = \frac{R^2}{D}\cdot\frac{4\alpha_1^7}{\pi^8\beta_1(1+\delta_{0n})}\cdot\frac{1}{(m^2+\gamma^2n^2)^4+m^4 l}. \tag{3.11}$$

The values $T_3(t_1)$ and $T_3(t_2)$ are determined from systems (3.9) during the inclusion of (3.6) and (2.7), if $j = 3$.

$$u_3(M) = p\iint_{(S)} v_{(3)3}(M, Q)\,dS_Q + \int_{(l_1)} T_3(t_1)\,v_{(3)3}(M, t_1)\,dl_{t_1} + \int_{(l_2)} T_3(t_2)\,v_{(3)3}(M, t_2)\,dl_{t_2}. \tag{3.12}$$

After substituting (3.10) in (3.12) we will obtain

$$u_3(M) = \frac{2Rp\alpha_1}{k}\sum_{m=1}^{\infty} C_{m0}\frac{1-(-1)^m}{m}\sin\frac{\pi m\alpha}{\alpha_1} - \tag{3.13}$$

$$-\frac{E_1 J_1}{2R^3}\sum_{m=1}^{\infty}\sum_{n=0}^{\infty} C_{mn}[b_m+(-1)^n a_m]\sin\frac{\pi m\alpha}{\alpha_1}\cos\frac{\pi n\beta}{\beta_1},$$

$$a_m = \int_0^{\alpha_1}\frac{\partial^4 u_3(t_1)}{\partial\gamma^4}\sin\frac{\pi m\gamma}{\alpha_1}\,d\gamma, \tag{3.14}$$

$$b_m = \int_0^{\alpha_1}\frac{\partial^4 u_3(t_2)}{\partial\gamma^4}\sin\frac{\pi m\gamma}{\alpha_1}\,d\gamma. \tag{3.15}$$

Having differentiated (3.13) by alpha four times, and multiplied the results by $\sin\frac{\pi m\alpha}{\alpha_1}$ and integrating by α from 0 to α_1 we will obtain on l_1 and l_2 respec-

tively

$$a_m = \frac{pR^2\alpha_1^2}{k}\left(\frac{\pi m}{\alpha_1}\right)^4 \frac{1-(-1)^m}{m} C_{m0} - \left(\frac{\pi m}{\alpha_1}\right)^4 \frac{E_1 I_1 \alpha_1}{4R^3} \sum_{n=0}^{\infty} C_{mn}[b_m + (-1)^n a_m]$$

$$b_m = \frac{pR^2\alpha_1^2}{k}\left(\frac{\pi m}{\alpha_1}\right)^4 \frac{1-(-1)^m}{m} C_{m0} - \left(\frac{\pi m}{\alpha_1}\right)^4 \frac{E_1 I_1 \alpha_1}{4R^3} \times \sum_{n=0}^{\infty} C_{mn}[a_m + (-1)^n b_m], \tag{3.16}$$

hence

$$a_m = b_m = \frac{pR^2\pi^4 m^3}{k\alpha_1^2} \cdot \frac{[1-(-1)^m]C_{m0}}{1 + \frac{I_1 E_1 \pi^4 m^4}{4R^3\alpha_1^3}\sum_{n=0}^{\infty} C_{mn}[1+(-1)^n]}. \tag{3.17}$$

Utilizing (2.15) and the static relationships of the theory of shells are known, it is possible to determine the sought for forces.

As it was to be anticipated, formulas for calculations, obtained as result of solving differential equations of a shell and found from integro-differential equations, are in conformity. The advantage of the second approach in solving the problem lies, apparently, in the fact, that it facilitates the selection of form of ideas of sought for displacements and unknown forces in places where the shell and ribs are jointed.

We like to point out, that the utilization of souble trigonometric series eliminates the necessity of determining arbitrary integration constants, it also improves the variation of rib charactersitics.

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